Two-Sample t-test Homogenous Variance

Course: Statistics 1

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Example

- Case:
  - Independent Samples: Same variance
  - Sample: Word Recall with and without mental images

Group 1 (images)	**Group 2 (no images)**
$N = 10$	$N = 10$
Mean, $M_1 = 26$	Mean, $M_2 = 18$
Std Dev, $S_1 = 4.71$	Std Dev, $S_2 = 4.22$
Variance, $S_1^2 = 22.22$	Variance, $S_2^2 = 17.78$
$\sum X = 260$	$\sum X = 180$
$\sum X^2 = 6960$	$\sum X^2 = 3400$

F Critical Value

- F_{cv} = 3.18 (*F*-Dist. Table, df_1 = 9, df_2 = 9, a = 0.05)

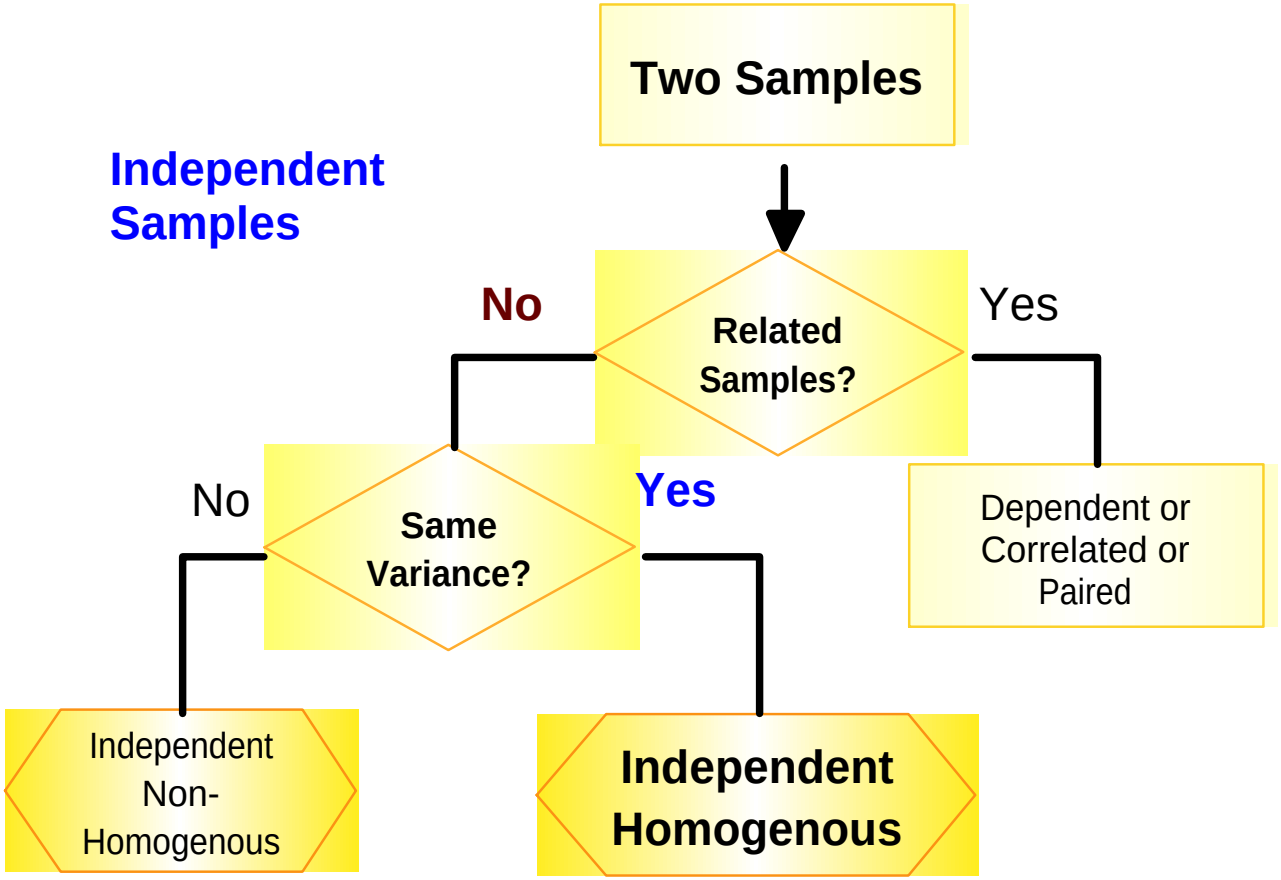
df 2	1	2	3	4	5	6	7	8	9	10	*df 2*
1	161.4462	199.4995	215.7067	224.5833	230.1604	233.9875	236.7669	238.8842	240.5432	241.8819	1
2	18.51276	19.00003	19.16419	19.24673	19.29629	19.32949	19.35314	19.37087	19.38474	19.39588	2
3	10.12796	9.552082	9.276619	9.117173	9.013434	8.940674	8.88673	8.845234	8.812322	8.785491	3
4	7.70865	6.944276	6.591392	6.388234	6.256073	6.163134	6.094211	6.041034	5.9988	5.964353	4
5	6.607877	5.786148	5.409447	5.192163	5.050339	4.950294	4.875858	4.818332	4.77246	4.735057	5
6	5.987374	5.143249	4.757055	4.533689	4.387374	4.283862	4.206669	4.146813	4.099007	4.059956	6
7	5.59146	4.737416	4.34683	4.120309	3.971522	3.865978	3.787051	3.725717	3.676675	3.636529	7
8	5.317645	4.458968	4.06618	3.837854	3.687504	3.580581	3.50046	3.438103	3.388124	3.347168	8
9	5.117357	4.256492	3.862539	3.63309	3.481659	3.373756	3.29274	3.229587	**3.18**	3.137274	9

Test for Homogeneity of Variance

- H_0: $s_1^2 = s_2^2$ So, H_a: $s_1^2 \neq s_2^2$
- F_{cv} = 3.18 ($df_1 = 9$, $df_2 = 9$ and a = 0.05)
- F_{stat} = 1.25 (22.22/17.78 = 1.25)
- Decision: Don't Reject H_0 that variances are same
  - Since $F_{stat} < F_{cv}$ *or* $1.25 < 3.18$

Conclusion: Variances are homogenous

Independent Samples with Homogenous Variance



Step 1: Hypotheses

- Null, H_0 (***no difference in means***)

  $\mu_1 - \mu_2 = 0$

- Alternative, H_a (Non-Directional)

  $\mu_1 - \mu_2 \neq 0$

Step 2: Set Rejection Criterion

- Significance Level: a = 0.05
- $df = n_1 + n_2 - 2 = 10 + 10 - 2 = 18$

- Critical value (*t*-distribution, *df* = 18)
  - Two-tailed (non-directional)
  - $t_{cv} = 2.101$
  - Reject H_0 if $t_{stat} >= 2.101$

Step 3: Compute Test Statistics

Given: $n_1 = 10, M_1 = 26, s_1 = 4.71, s_1^2 = 22.22, \Sigma X = 260, \Sigma X^2 = 6960$

Given: $n_2 = 10, M_2 = 18, s_1 = 4.22, s_1^2 = 17.78, \Sigma X = 180, \Sigma X^2 = 3400$

$$s^2 = \frac{[\sum X_1^2 - \frac{(\sum X_1)^2}{n_1}] + [\sum X_2^2 - \frac{(\sum X_2)^2}{n_2}]}{n_1 + n_2 - 2} = \frac{[6960 - \frac{67600}{10}] + [3400 - \frac{32400}{10}]}{10 - 10 - 2} = 20$$

Std Error: $s_{(M_1 - M_2)} = \sqrt{s^2(\frac{1}{n_1} + \frac{1}{n_2})} = \sqrt{20(\frac{1}{10} + \frac{1}{10})} = 2$

Test statistics, $t = \frac{M_1 - M_2}{s_{(M_1 - M_2)}} = \frac{26 - 18}{2} = 4$

Step 4: Confidence Interval

- CI = Statistics +/- Critical Value (Standard Error)
- Mean Difference, M_D = 8, Since 26 – 18
- t_{cv} = 2.101 (two-tailed, *df* = 18 and a = 0.05)

- CI_{95} = 8 +/- 2.101(2) = 3.798 **to** 12.20

Step 5: Effect Size

- ES = Mean Difference / Standard Error = $(M_1 - M_2)/s$
- Calculated s^2 (pooled estimate) = 20
- So **s** = Sqrt(20) = 4.47

- ES = (26 – 18)/4.47 = 8/4/47 = 1.79

- **Conclusion**: A very large treatment effect (> 0.8)
  - Group using mental images recalled significantly more words than the group with no images

Step 6: Decision

- Met homogeneity of Variance assumption
- Reject H_0:
  - 1. $t_{stat} > t_{cv}$ or $4 > 2.101$
  - *2. Hypothesized population difference of 0 is Outside* CI_{95}
    - CI_{95}: *3.80 to 12.20*
  - 3. ES = 1.79 > 0.8, is large
- **Conclusion**: The group using mental images recalled significantly more words than the group with no images

SPSS Outputs

Group Statistics

	Group2	N	Mean	Std. Deviation	Std. Error Mean
Words	1.00	10	26.0000	4.71405	1.49071
	2.00	10	18.0000	4.21637	1.33333

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Words	Equal variances assumed	.384	.543	4.000	18	.001	8.00000	2.00000	3.79816	12.20184
	Equal variances not assumed			4.000	17.780	.001	8.00000	2.00000	3.79443	12.20557

***F*-Test: <u>Do not</u>** reject null hypothesis - assume same variance, since F_{sig} = **0.543** > **0.05**

***t*-Test: Reject** null hypothesis – so means are not same, since:

1. t_{test} = **4.0** > t_{cv} = **2.101** (two-tailed, *df* = 18, a = 0.05)

2. *p*-value = **0.001** < **0.05** and **3.** CI_{95} **<u>Do not</u>** contains **0**